Errors and uncertainties

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Errors and uncertainty of measurement

- Metrology – the science on accuracy and precision of measurement.
- Results of measurement may fluctuate and differ from true value.
  - The fluctuation may be caused by instrumentations as well as the properties of measured signal (noise, disturbances) = stochastic error.
  - The difference may be caused mainly but not only inaccurate properties of instrument (electronic circuits, mechanical parts, chemical sensors, …) = systematic errors
  - Gross errors (!have to be always corrected and removed) – caused for example by wrong instrument, broken instrument, wrong reading, wrong calculation, etc.
- We do not know the difference between the true value and results of measurement
- We can only estimate the true value and the difference with some probability – we are not sure what is the true.
- That is why we are speaking about uncertainty of measurement

Errors and uncertainty of measurement

- Uncertainty u is expressed in the form of interval around the estimation of true value x_{est}, where the true value x_{TRUE} lays with some probability $P_{required}$

$$P(x_{est} - u < x_{TRUE} < x_{est} + u) = P_{required}$$

- x_{est} and u are needed to be estimated from measurement (measurement results, statistical behavior of results, employed instrumentations, measurement stand, etc.)
- The interval may be also unsymmetrical (given by the distribution of errors)
  - The most common distribution – Gaussian and uniform (the interval is symmetrical)

Uncertainty type A

- Caused by stochastic errors – u_a is calculated from repeated measurement by statistical approach.
- The true value is usually estimated as the mean value calculated from *N* repeated measurements $$\bar{x} = \frac{\sum_{i=1}^{N} x_i}{N}$$
- Standard uncertainty u_a is calculated as the standard deviation of mean value (the probability is 0,68 for Gaussian distribution) $$u_A = s_{\bar{x}} = \sqrt{\frac{\sum_{i=1}^{N}(x_i - \bar{x})^2}{N(N-1)}}$$
- Note: the standard uncertainty u_a for single measurement is: $$u_A = s_{x_i} = \sqrt{\frac{\sum_{i=1}^{N}(x_i - \bar{x})^2}{(N-1)}}$$

Uncertainty type B

- Uncertainty type B is any uncertainty calculated by a different method than statistic:
  - results from previous measurements,
  - general or empirical knowledge of the behavior of the instruments used,
  - specifications of the manufacturer,
  - calibration certificates,
  - the uncertainty attributed to reference quantity values mentioned in studies, textbooks or norms.

Systematic error and uncertainty type B

- The most common and simple way is calculation from instrument accuracy (max. inaccuracy $\Delta_{max}x$) specified by producer (see instrument datasheet)
  - Analog instrumentations: $\Delta_{max}x$ is given in datasheet as a percentage δ_M from the applied measurement range *M*: $$|\Delta_{\max} x| \le \frac{\delta_M}{100} M$$
  - Digital instrumentation: sum of 2 components
    - Percentage δ_x from the result of measurement *x*
    - The component related to the applied measurement range expressed usually as number of digits K multiplied by the weight of one on the least order of instrument display m_1

$$|\Delta_{\max} x| \le \frac{\delta_x}{100} x + K.m_1$$

- Calculation of standard uncertainty u_B: (for the most probable uniform distribution) $$u_B = \sigma = \frac{|\Delta_{\max} x|}{\sqrt{3}}$$

Combined uncertainty and propagation of u

- Combined uncertainty u_c = the total uncertainty of measurement

$$u_c = \sqrt{u_a^2 + u_b^2}$$

- If result y is calculated as a combination (function f(.)) from N particular measurements a_i of different quantities

$$y = f(a_1, a_2, \dots a_N)$$

- Then the standard uncertainty of y is: $$u_y = \sqrt{\sum_{i=1}^{N} \left(\frac{\partial f(a_1, a_2, \dots, a_N)}{\partial a_i} \right)^2 u_{a_i}^2}$$
- Example: *R*=*V*/*I*, $$u_y = \sqrt{\left(\frac{\partial R}{\partial V}\right)^2 u_V^2 + \left(\frac{\partial R}{\partial I}\right)^2 u_I^2} = \sqrt{\left(\frac{1}{I}\right)^2 u_V^2 + \left(\frac{V}{I^2}\right)^2 u_I^2}$$

Expanded uncertainty U

- Expanded uncertainty *U* is calculated by multiplication of standard uncertainty *u* by the coefficient *k, U=ku*
- *K* is chosen according to required probability that the true values of measured quantity lays with the interval specified by the expanded uncertainty
  - Examples for Gaussian distribution
    - k=1, p=0,68 (standard uncertainty)
    - k=3, p=0.997
    - k=2, p=0.99
    - k=2/3, p=0.5

Calculation example

- Multimeter Agilent 34405A,
  - measurement of DC voltage, 1V range
  - Condition of measurement according to requirements given by producer
  - Multimeter datasheet: max error = 0.025+0.006 (in percentage from measurement and from range)
- Acquired values:

x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	x_9	x_{10}
0,565V	0,561V	0,564V	0,563V	0,567V	0,566V	0,562V	0,564V	0,568V	0,564V

Calculation

- $\bar{x} = 0{,}5644\text{V}$ $\quad u_A = s_{\bar{x}} = 0{,}000686\text{V}$
- $\Delta_{max}x$=(0,00025.0,5644V+0,00006.1V)=0,0002011V.
- For uniform distribution of systematic errors:

$$u_B = 0{,}0002011\text{V} / \sqrt{3} = 0{,}000116\text{V}$$

- Combine standard uncertainty:

$$u_C = \sqrt{u_A^2 + u_B^2} = \sqrt{(0{,}000686\text{V})^2 + (0{,}000116\text{V})^2} = 0{,}000696\text{V}$$

- The result of measurement:

$$x_{TRUE} \in [0{,}5644\text{V} \pm 0{,}0007\text{V}]$$

Demonstrations

- Simulated multimeter
- myDAQ multimeter with statistics