# Errors and uncertainties

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#### Errors and uncertainty of measurement

- Metrology the science on accuracy and precision of measurement. .
  - Results of measurement may fluctuate and differ from true value. The fluctuation may be caused by instrumentations as well as the properties of measured signal (noise, disturbances) = stochastic error.
  - signal (noise, usual uces) succitasia error. The difference array be caused mainly but not only inaccurate properties of instrument (electronic circuits, mechanical parts, chemical sensors, ...) = systematic errors Gross errors ((Thave to be always corrected and removed) caused for example by wrong instrument, broken instrument, wrong reading, wrong calculation, etc. a.
  - We do not know the difference between the true value and results of
- measurement We can only estimate the true value and the difference with some
- probability we are not sure what is the true. That is why we are speaking about uncertainty of measurement

#### Errors and uncertainty of measurement

Uncertainty u is expressed in the form of interval around the estimation of . true value  $x_{est}$ , where the true value  $x_{TRUE}$  lays with some probability  $P_{required}$ 

 $P(x_{est} - u < x_{TRUE} < x_{est} + u) = P_{required}$ 

- $\mathbf{x}_{\mathrm{est}}$  and  $\mathbf{u}$  are needed to be estimated from measurement (measurement results, statistical behavior of results, employed instrumentations, measurement stand, etc.)
- The interval may be also unsymmetrical (given by the distribution of errors) The most common distribution – Gaussian and uniform (the interval is symmetrical)

### Uncertainty type A

- Caused by stochastic errors u<sub>a</sub> is calculated from Caused by stochastic ends  $x_{u}$  is causate  $x_{u}$  is repeated measurement by statistical approach. The true value is usually estimated as the mean value  $\frac{1}{x} = \frac{\sum_{i=1}^{N} x_i}{N}$
- . calculated from N repeated measurements
- Standard uncertainty  $u_a$  is calculated as the standard  $u_A = s_x = \sqrt{\frac{\sum_{i=1}^{N} (x_i \overline{x})^2}{N(N-1)}}$ . Gaussian distribution)

 $u_{A} = s_{x_{i}} = \sqrt{\frac{\sum_{i=1}^{N} (x_{i} - \overline{x})^{2}}{(N-1)}}$ Note: the standard uncertainty  $u_{a}$  for single measurement is:

### Uncertainty type B

- Uncertainty type B is any uncertainty calculated by a different method than statistic:
  - results from previous measurements,
  - a general or empirical knowledge of the behavior of the instruments used, specifications of the manufacturer,
  - calibration certificates.
  - the uncertainty attributed to reference quantity values mentioned in studies, textbooks or norms.

## Systematic error and uncertainty type B

• The most common and simple way is calculation from instrument accuracy (max. inaccuracy  $\Delta_{max}$ ) specified by producer (see instrument datasheet) • Analog instrumentations:  $\Delta_{max}$  is given in datasheet as a percentage  $\delta_M$  from the applied measurement range M:  $\left|\Delta_{\max} x\right| \leq \frac{\delta_M}{100} M$ 

- Digital instrumentation: sum of 2 components Percentage  $\delta_x$  from the result of measurement x .
  - The component related to the applied measurement range expressed usually as number of digits K multiplied by the weight of one on the least order of instrument display  $m_1$

$$\left|\Delta_{\max} x\right| \leq \frac{\sigma_x}{100} x + K.m$$

 $|\Delta_{\max}x| \simeq \frac{1}{100}x + \dots + \frac{1}{100}x$ Calculation of standard uncertainty  $u_{\text{B}}$ : (for the most probable uniform distribution)  $u_{B} = \sigma = \frac{|\Delta_{\max}x|}{\sqrt{3}}$ 



- Combined uncertainty  $u_c$  = the total uncertainty of measurement  $u_c = \sqrt{u_a^2 + u_b^2}$
- If result y is calculated as a combination (function f(..)) from N particular measurements a<sub>i</sub> of different quantities

 $y = f(a_1, a_2, \dots a_N)$ 

• Then the standard uncertainty of y is:  $u_y = \sqrt{\sum_{i=1}^{N} \left(\frac{\partial f(a_i, a_2, \dots, a_N)}{\partial a_i}\right)^2 u_{a_i}^2}$ 

Example: R=V/I,  $u_{y} = \sqrt{\left(\frac{\partial R}{\partial V}\right)^{2} u_{V}^{2} + \left(\frac{\partial R}{\partial I}\right)^{2} u_{I}^{2}} = \sqrt{\left(\frac{1}{I}\right)^{2} u_{V}^{2} + \left(\frac{V}{I^{2}}\right)^{2} u_{I}^{2}}$ 

## Expanded uncertainty U

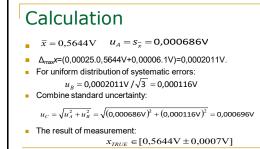
- Expanded uncertainty U is calculated by multiplication of standard uncertainty u by the coefficient k, U=ku
- K is chosen according to required probability that the true values of measured quantity lays with the interval specified by the expanded uncertainty
  - Examples for Gaussian distribution
    k=1, p=0,68 (standard uncertainty)
    - k=3, p=0.997
    - k=2, p=0.99
    - k=2/3, p=0.5

### Calculation example

#### Multimeter Agilent 34405A,

- measurement of DC voltage, 1V range
- Condition of measurement according to requirements given by producer
  Multimeter datasheet: max error = 0.025+0.006 (in percentage from measurement
- Multimeter datasheet: max error = 0.025+0.006 (in percentage from measuremen and from range)
- Acquired values:

	x3	<i>x</i> <sub>4</sub>	x <sub>5</sub>	x <sub>5</sub>	X7	×s	x <sub>9</sub>	x10
0,561V	0,564V	0,563V	0,567V	0,566V	0,562V	0,564V	0,568V	0,564V
0	.561V	.561V 0,564V	.561V 0,564V 0,563V	.561V 0,564V 0,563V 0,567V	.561V 0,564V 0,563V 0,567V 0,566V	.561V 0,564V 0,563V 0,567V 0,566V 0,562V	561V 0,564V 0,563V 0,567V 0,566V 0,562V 0,564V	561V 0,564V 0,563V 0,567V 0,566V 0,562V 0,564V 0,568V



# Demonstrations

- Simulated multimeter
- myDAQ multimeter with statistics