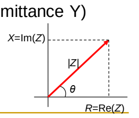
Introduction to signals and systems, theory of measurements

Self study - repetition

- Study from the textbook:
  - Electrical current I and charge Q
    - Electrical current: movement of electrical charges (positive in convention, negative = electrons in reality)
    - The basic units: ampere (current), coulomb (charge)
  - Electrical voltage V, U
    - Ability to move charges (difference of potentials)
    - The basic unit: volt
  - Resistance R
    - the property of material determining relations between constant voltage and current
    - The basic unit ohm
  - Relation V,I and R: Ohm law $R = \frac{V}{I}$

Self study - repetition

- Capacitance
  - The ability to accumulate energy in electric field (accumulate charge) $i(t) = C\frac{\mathrm{d}v}{\mathrm{d}t}$
  - The basic unit: farad
- Inductance
  - The ability to accumulate energy in magnetic field $v(t) = L\frac{\mathrm{d}i}{\mathrm{d}t}$
  - The basis unit: henry
- Complex imitance (expressed as impedance Z or admittance Y)
  - Combination of resistance, capacitance and inductance
  - The basic unit: ohm (or siemens)
  - $Z(\mathrm{j}\omega)=R(\omega)+\mathrm{j}X(\omega)$, $Y(\mathrm{j}\,\omega)=G(\omega)+\mathrm{j}B(\omega)$



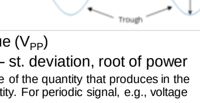
Self study – power and energy

- Power – change of energy in time, the rate of change of energy
- Energy – ability to perform a work: joule = Ws
- DC voltage and current $P = U \cdot I = I^2 \cdot R = \frac{U^2}{R}$
- Immediate power $p(t) = \frac{\mathrm{d}E(t)}{\mathrm{d}t}$
- The total power $P_{tot} = \frac{1}{T}\int_0^T u(t) \cdot i(t)\mathrm{d}t$
- Parseval theorem $P_{tot} = \sum_{i=0}^{N} P_i = \frac{1}{R}\sum_{i=0}^{N} U_i^2$
- The basic unit: watt; 1 watt is the power, which change the energy by 1joule in 1 second

Signals

- Abstraction of behavior of voltage, current, EM field, …
  - Constant in time – abbreviation DC and capital letters
  - Variable in time – abbreviation AC and small letters (AC is used mostly for sinewave with relatively low frequency in power electronics)
- Deterministic – we know the values in the future and past
  - Periodic signals repeat their course after a time interval = period T, $x(t+k.T)=x(t)$, k is an integer number
- Stochastic (random) – we cannot calculate their value in a future, we can only describe their behavior by statistical parameters
  - Mean value (DC component, average value)
  - Standard deviation (effective = rms value= root from dispersion)
  - Dispersion (power)
  - Etc.

Parameters



- Time: period, rise time, fall time, with, ….
- Magnitude
  - Mean value – DC, average
  - Amplitude, peak value (V_P), peak-to-peak value (V_{PP})
  - Effective value (RMS – bounded with power) – st. deviation, root of power
    - Effective value has been defined as equivalent dc value of the quantity that produces in the load the same heating effect as the measured ac quantity. For periodic signal, e.g., voltage *v*(*t*), with period *T* the effective value V_{rms} is:

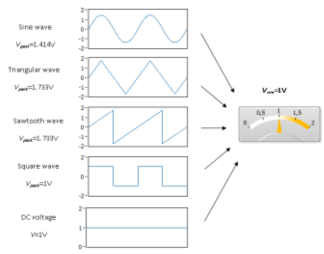
$$P = \frac{1}{TR}\int_0^T v_{AC}^2(t)dt = \frac{V_{DC}^2}{R} = \frac{V_{rms}^2}{R} \qquad V_{rms} = \sqrt{\frac{1}{T}\int_0^T v^2(t)\mathrm{d}t} = \sqrt{\mathrm{mean}\left(v^2(t)\right)}$$

  - Power
    - In signal theory the load R has the unit value or the nominal value, e.g., 50ohms

Examples

The abbreviation RMS or sometimes TRMS comes from form of defining equation “Root Mean Square” or “True Root Mean Square”.

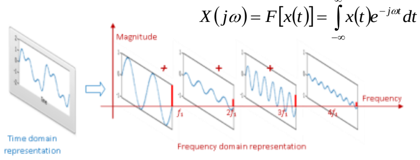
RMS value of time varying quantity depends not on its amplitude about also on shape of quantity time variation (waveform)!!!



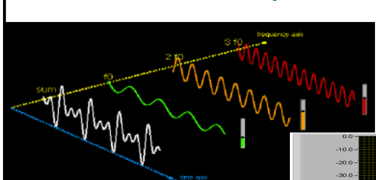
Representation in frequency domain (spectrum)

- The spectrum describes distribution of the quantity to its frequency components (harmonics) and it can be calculated by applying Fourier transformation ($\omega=2\pi f$ is the angular (circular) frequency):

$$X(j\omega) = F[x(t)] = \int_{-\infty}^{\infty} x(t) e^{-j\omega t} dt = X(\omega) e^{j\Theta(\omega)},$$

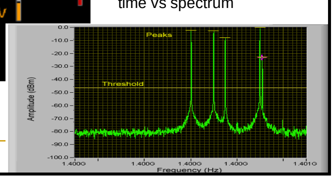


Zobrazenie spektra

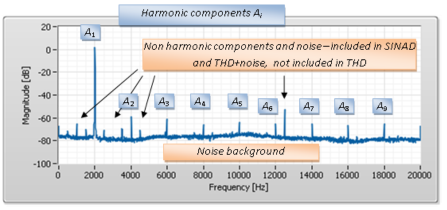


Demonstration 1 - view

Demonštrácia 2 – time vs spectrum



Example of spectrum



Decibels

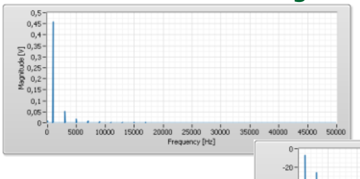
- dB (V, A, W, …)

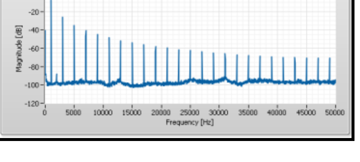
$$P_{dB} = 10\log\frac{P}{P_{ref}} = 10\log\frac{V^2/R_Z}{V_{ref}^2/R_Z} = 20\log\frac{V}{V_{ref}}$$

$$P_{dB} = 10\log\frac{P}{P_{ref}} = 20\log\frac{I\cdot R_Z}{I_{ref}\cdot R_Z} = 20\log\frac{I}{I_{ref}}$$

- Change by 3dB = doubled power or 1,4.V_1
- Change by 6dB = quaded power and doubled voltage
- Change by 10dB, 20dB, -3dB, -6dB, ... ?
- dB is often with some modifier P, PP, RMS, m, c, ...
- dB is often used description of magnitude in spectrum
- %, °%, ppm – parts per million
- ...

Presentation of magnitude spectrum





Digitizing Signal

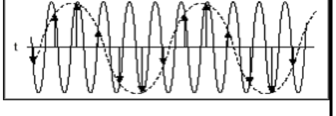
- Conversion from analog value, which can reach any value to an integer number, which may have only a value from a finite set of numbers.
- Digitizing has two steps:
  - Sampling = taking and holding immediate value of analog signal.
    - The process is controlled by clock with sampling rate (frequency)
  - Quantization = conversion of sample to number (by analog to digital convertor
    - The difference of analog values represented by two adjacent values is ADC quantization step
    - The error caused by rounding the values within quantization step to unique digital value (integer number) is quantization noise.

Sampling

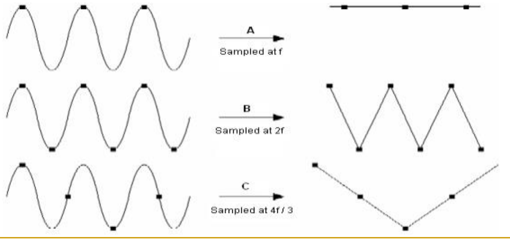
- Shannon condition (Nyquist theorem) $f_S \geq 2f_{\max}$
- Usually more than 2 – oversampling
- Aliasing = compents in signal spectrum with frequencies higher than $f_S/2$ (Nyquist frequency) will be transformed into new lower frequencies under Nyquist frequency) f_{alias}

$$f_{alias} = abs\left(nearest(kf_S) - f_i\right)$$

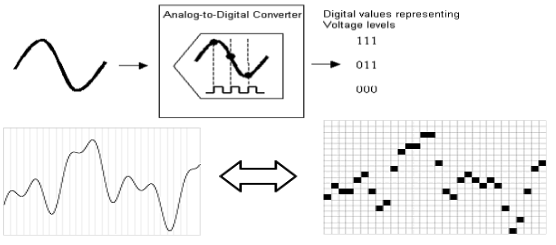
Demonstartion



Sampling



Digitizing -quantization



Quantization

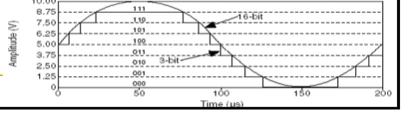
- ADC quantization step is the smallest detectable change of input signal from ADC out codes = the difference of analog values represented by two adjacent values is ADC quantization step

$$\Delta = {V_{FS}}/{2^N - 1} \cong {V_{FS}}/{2^N}$$

- ADC Full scale are limits (maximum and minimum) of input signal, which can be correctly converted into a number (input range of ADC)
- Quantization noise - the error caused by rounding the analog values within quantization step to unique digital value (integer number)

$$\varepsilon_{rms} \cong {\Delta}/{\sqrt{12}}$$

Simulation



Spectrum of ditized signal

- Spectrum can be calculated also from digitized signal by Discrete Fourier Transformation (DFT, its fast implementation is Fast Fourier Transformation FFT) $X(2\pi k\Delta f) = X_k = \sum_{i=0}^{n-1} x(i\Delta t) e^{\frac{-j2\pi ki}{n\Delta t}} = \sum_{i=0}^{n-1} x_i e^{\frac{-j2\pi f_s ki}{n}}$

  n - number of samples, $\Delta t = 1/f_S$ – period of sampling.
- Results: $n/2$ complex numbers – samples of spectrum with frequency distance of $\Delta f = f_S/n$ (frequency resolution)
- !!! DFT spectrum is only an approximation of true signal spectrum!!!
- The main error is caused by leakage effect.
  - Windowing may suppress the leakage effect

$$X(2\pi k\Delta f) = \sum_{i=0}^{n-1} w(i\Delta t) x(i\Delta t) e^{\frac{-j2\pi ki}{n\Delta t}}$$

simulation

Definitions of some signal parameters

- Total harmonic distortion - THD (the difference of distorted sinewave from mathematical sinewave)

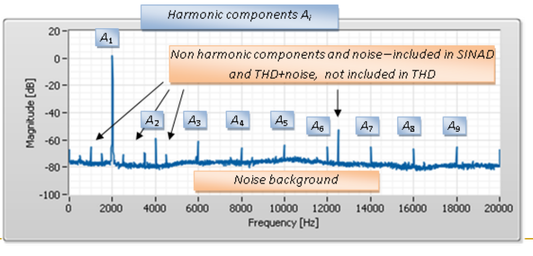
$$THD_{dB} = 10\log\frac{\sum_{h=2}^{N} P_h}{P_1} = 20\log\frac{\sqrt{\sum_{h=2}^{N} A_h^2}}{A_1},$$

  - P_h resp. A_h are powers, resp. rms value of harmonics in signal spectrum
- Signal to Noise and Distortion ratio – *SINAD*

$$SINAD_{dB} = 10\log\frac{P_1}{\eta_{rms}^2} = 20\log\frac{A_{1rms}}{\eta_{rms}} = -THD + noise_{dB}$$

  - P_1 resp. A_{1rms} is the power, resp. rms value of basic harmonics and η_{rms} is the rms value of the total nose and distortion.

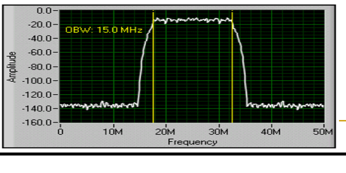
Typical spectrum of periodic signal



Two port circuit characterisation

- Frequency response $H(j\omega)$ – the ratio of spectra of signals on output and input of two port circuits (e.g. amplifier, filter, etc.)
  - Often expressed in the form of magnitude $A(\omega)$ and phase $\Theta(\omega)$ response

$$H(j\omega) = \frac{X_o(j\omega)}{X_i(j\omega)} = A(\omega)e^{-j\Theta(\omega)}, \quad A(\omega) = \frac{|X_o(j\omega)|}{|X_i(j\omega)|}, \quad \Theta(\omega) = \Theta_o(\omega) - \Theta_i(\omega)$$



Derived parameters

- Bandwidth: the difference between the maximal and minimal frequency, at which magnitude response decreases by 3dB.
- Relative bandwidth: the ration of bandwidth to the central frequency.
- Gain if $A(\omega)>1$, attenuation if $A(\omega)<1$

Príklad AFCH