Instrumentation for spectral measurements

Ján Šaliga
2017

Spectrum

- Substitution of waveform by the sum of harmonics (sinewaves) with specific amplitudes, frequences and phases. The sum of sinewave have the same waveform in the as the origin signal
- Measurement is spectrum allows better understanding some signals and ciruit behavior than time analysis, e.g. different distortion, analysis and determination of modulated signal, noise and jitter characteristics, etc.
- The most common expression of spectrum:
  - Magnitude (and phase - rarely) – magnitudes of spectral components
  - Power – power of spectral components
  - Power spectral density – power related to the bandwidth of 1Hz
- Magnitudes are usually expressed in dBc (ratio to the basic harmonics or carrier for modulated signals)

The basic idea of measurement

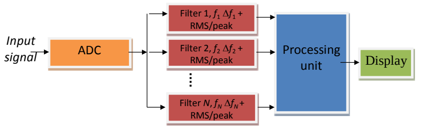
- Instruments must divide the bandwidth of interest to narrow bands and measure voltage (power) in each band
- How to do it?
  - Narrow bandpass filter
    - Central frequency must be tunable within required frequency range of interest (span)
    - Filter bandwidth and quality are usually required to be setable and constant while tuning
  - Fourier transformation (DFT from digitized samples)

Analog filters (LC, RLC, …)

- Not proper for common measurement
  - Filter bandwidth B changes with central frequency f_c tuning (the absolute bandwidth B is not constant because quality of filter Q is constant

$$\Delta = \frac{B}{f_C} = \frac{f_{\max} - f_{\min}}{f_C} = \frac{1}{Q} \Rightarrow B = \frac{f_C}{Q} \neq const.$$

- Equivalent digital filters are used for some acoustic measurements



FFT (DFT) analyzer

- Based on digital signal processing:
  - Measured signal is digitized and recorded in memory (N samples)
  - Spectrum is calculated from the record by DFT (FFT)

$$X'(k\Delta f) = \frac{1}{N}\sum_{n=0}^{N-1} x(nT_s)e^{-j2\pi k\Delta f nT_s}, \quad k = 0, 1, \ldots, N-1$$

  - The achieved results are usually normalized to be in conformity with real physical world (range of frequencies from) to $0{,}5f_s$

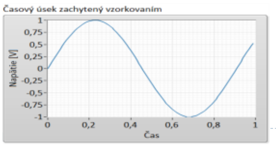
$$X(k\Delta f) = \begin{cases} X'(0) \\ 2X'(k\Delta f) \end{cases}, \quad k = 0, 1, \ldots, \left\lfloor \frac{N}{2} - 1 \right\rfloor$$

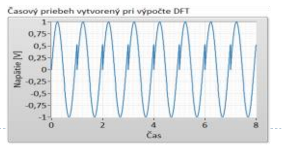
Errors in DFT spectrum I.

- SDFT spectrum is discrete (a final number of samples of real spectrum, e.e., the spectrum components are known only for frequencies with step $\Delta f = f_S/N$ (frequency resolution).
- Consequence:
  - Components with frequency difference lower than frequency resolution can not be separated and their energy is summarized into one common peak in DFT spectrum
  - Improving of frequency resolution:
    - Increasing number of samples in record – limited by memory and time needed for calculation of DFT
    - Decreasing sampling frequency – limited by Shannon condition

Errors in DFT spectrum II.

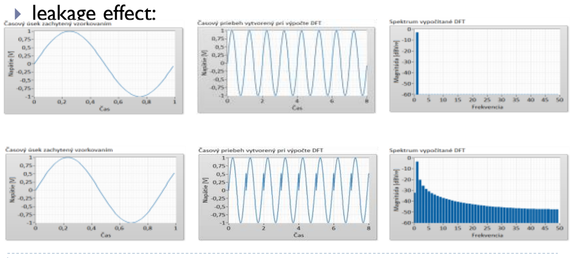
- Spectrum is calculated from a time limited segment of real signal
- The formula spread the segment into infinity signal by periodic repetition of the segment
- Consequence
  - Spectrum is calculated from virtual signal different from the measured signal, e.g., periodic signal acquired within time interval different from integer multiply of period
  - Leakage effect





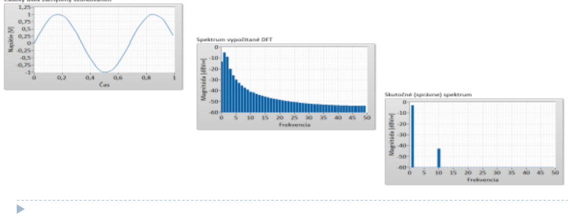
Errors in DFT spectrum III.

- leakage effect:



Consequence of leakage effect

- Component with frequency close to other component and much smaller amplitude can be hidden by leakage effect



Suppression of leakage effect

- Windowing
  - Window function is applied on record befor DFT

$$X'(k\Delta f) = \sum_{n=0}^{N-1} w(nT_s).x(nT_s)e^{-j2\pi k\Delta f nT_s}$$

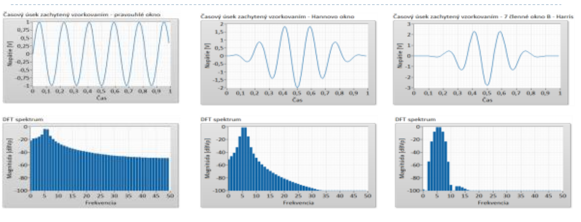
  - Window functions: the samples at the beginning and end of record are continually decreased to 0 from center of record.
  - The often used windows: cosine windows

$$w(t) = \sum_{n=0}^{\infty} a_n \cos\frac{2\pi nt}{T} \qquad -\frac{T}{2} < t < \frac{T}{2}$$

    - A finite number of coefficients (a_n from a chosen n are zeros)
    - The most used cosine window: Hann window

$$w(t) = 0.5 - 0.5\cos\frac{2\pi t}{T} \qquad -\frac{T}{2} < t < \frac{T}{2} \qquad w(nT_S) = 0.5\left(1 - \cos\frac{2\pi nT_S}{T}\right)$$

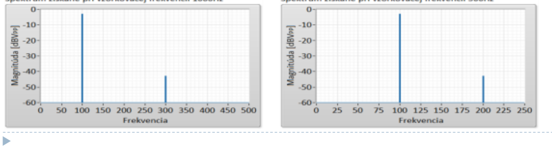
Different windows



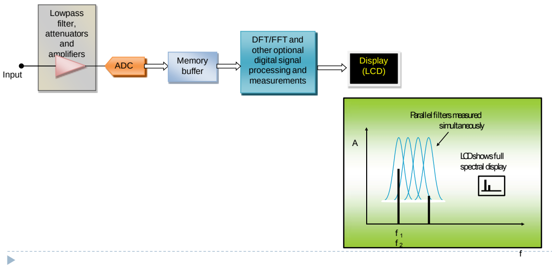
- Windows suppress leakage but causes additional magnitude error

Aliasing

- DFT requires suppression of components in measured signal with frequencies higher than Nyquist frequency by input antialiasing low pass filter otherwise components with incorrect position in frequency can be found in calculated spectrum
- Example:



FFT analysers



Noise floor

- A noise is present in any real signal
- Noise is produced by any electronic circuit including measurement instrumentation
- The noise is also produced during quantization
- The most common noise is more less white = the power is distributed uniformly over a frequency range (power spectral density is constant)
- Noise floor (level) can mask small spectral components (they can be hidden in the noise)
- Because of uniformed power distribution noise floor can be decreased by narrowing filtration, i.e. increasing frequency resolution improve detection of small spectral components hidden in noise

$$nf\_change = 10\log\frac{BW_2}{BW_1}$$

Heterodyne filtration

- The idea: to transpose frequency of measured spectral component (downconverter) to the frequency of fixed filter ω_{if} (intermediate filter) and this way to avoid tuning narrowband filter
- Transposing can be performed by mixing measured spectral component (ω_i, measured signal) with generated sinewave (ω_o, local oscillator in receiver)
  - Mixing = multiplication.

$$\cos(\omega_o t)\times A_i\sin(\omega_i t) = \frac{A_i}{2}\left[\sin((\omega_o-\omega_i)t)+\sin((\omega_o+\omega_i)t)\right]$$

- If ω_o-ω_i=ω_{if}, the component with frequency ω_i afrer transposition to the new frequency ω_{if} separated from other components in signal spectrum and transferred by filter for following processing

One stage mixing and filtration

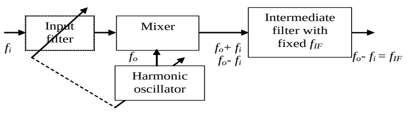


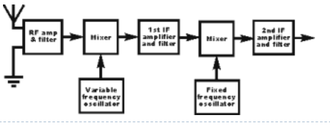
Image frequencies: input spectral components with frequency f_i+2f_{IF} are not rejected by the basic principle

$$\omega_{im} = \omega_i + 2\omega_{IF} = \omega_o + \omega_{IF} \rightarrow \omega_o - \omega_{im} = -\omega_{IF}$$

The consequence: a tunable input filter rejecting spectral components with image frequency = with bandwidth < 2ω_{IF} is needed.

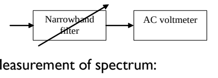
Multistage mixing and filtering

- Narrowband analog filters are easily realizable at low frequencies (f_{IM} is low)
  - But if f_{IM} is low then image frequency is very close to required input frequency ⇒ input tunable filter must have extremely selective = difficult realization
- Solution: multistage mixing and filtering
  - Step by step mixing (transposing down) and more and more narrowed filtering
  - Only the first stage in tuned (to choose frequency component for processing), the rest works with fixed frequencies (the only task is to increase selectivity of the all chain - narrowing bandwidth

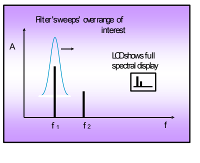


Selectiv voltmeter (obsolete)

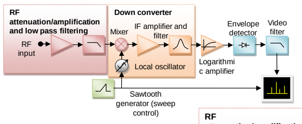
- Combination of narrowband filtration and AC voltmeter



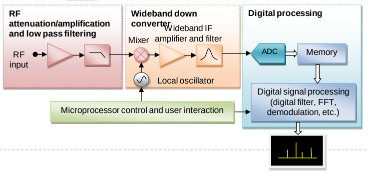
- Measurement of spectrum:
  - Only on a few chosen frequencies
  - Step by step tuning and measurement - time consuming and not much practical
    - Improvement: automated tuning and displaying measured values on the screen (swept-tuned spectrum analyzer)



Swept tuned spectrum analyzer



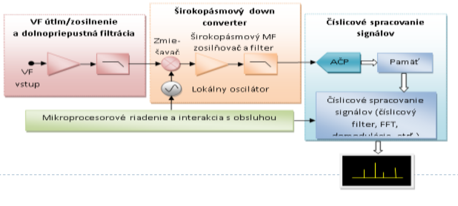
- Analog signal processing (obsolete)



- Digital signal processing

Vektorový signálový analyzátor

- Kombinuje heterodynný princíp s FFT analýzou a ďalšími číslicovými metódami spracovania a merania signálov:
  - Frekvenčne sa transponuje pomerne široké pásmo z vf na nf, kde sa digitalizuje rýchlym ADC a následne matematicky spracuje podľa požiadavky obsluhy



Basic measurements in spectrum

Distortion measurements

- Basic analysis of spectrum:
  - Identification of components
    - Wanted (components - magnitudes, position = frequencies, spectrum bandwidth, …)
    - Spurious components and noise (identification of source, frequencies, magnitudes power, …)
- Numerical expression of distortion
  - **Total harmonic distortion** (the difference between mathematical sinewave (the basic harmonics) and real distorted signal

$$THD = \sqrt{\sum_{h=2}^{m} A_h^2} \Big/ A_1 \quad THD = \sqrt{\sum_{h=2}^{m} A_h^2} \Big/ \sqrt{\sum_{h=1}^{m} A_h^2} \quad THD_{dB} = 20 \log THD$$

$$THD + noise = rms(signal - bas.harm.)/rms(signal)$$

  - **Intermodulation distortion** - distortion by nonharmonic components produces usually by nonlinearity of electronic circuit processing signal consisting of two

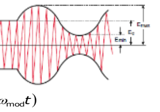
$$IMD_{dB} = 10 \log \frac{P_{intermod}}{P_{signal}} = -10 \log \frac{\min\{Y(J_1), Y(J_2)\}}{\max_{|j|,|l| \ge 1,\ j+l=q \ge 1} \{Y(jJ_1 + lJ_2)\}}$$

Amplitude modulation

- LF modulation signal (information) modulate (control) amplitude of carrier RF sinewave

$$u_{carrier}(t) = U_{carrier} \sin(\omega_{carrier} t)$$

$$u_{AM}(t) = U_{carrier} \left(1 + m.\frac{f_{mod}(t)}{\max|f_{mod}(t)|}\right) \sin(\omega_{carrier} t) = U_{carrier}(1 + m.f(t)) \sin(\omega_{carrier} t) \quad |f(t)| \le 1$$

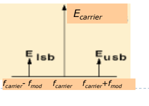


m is the modulation index (m<1for common basic AM).

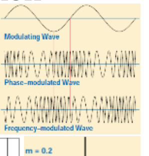
- Testing AM modulator: unknown measured parameter is m
  - Test signal: sinewave with a nominal amplitude, e.g. ,1: $u_{mod}(t) = \sin(\omega_{mod} t)$
- Then the AM spectrum is:

$$u_{AM}(t) = U_{carrier} \left[\sin(\omega_{carrier} t) + \frac{m}{2} \sin((\omega_{carrier} - \omega_{mod})t) + \frac{m}{2} \sin((\omega_{carrier} + \omega_{mod})t) \right]$$

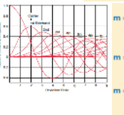
$$m = \frac{2E_{lsb}}{U_{carrier}} = \frac{2E_{usb}}{U_{carrier}} = \frac{E_{lsb} + E_{usb}}{U_{carrier}} \qquad m[\mathrm{dB}] = A_{SB}[\mathrm{dB}] - A_C[\mathrm{dB}] + 6\mathrm{dB}$$

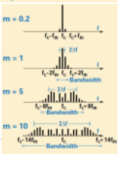


Angular and frequency modulation



- Angular and frequency modulation are very similar - modulating signal control instantaneous frequency of phase on carrier sinewave
- More complex spectrum than AM (many components)
  - Amplitude components are given by Bessel functions of argument m (m is the index of FM modulation, **m=peak frequency deviation/modulation frequency** or **m=peak phase deviation in radians**





  - Measurements:
    - Carrier (central frequency)
    - Real bandwidth
    - Monitoring of transmitter

Mathematical background of FM and PM

- Carrier: $u_c(t) = U_c \sin(\omega_c t) = U_c \sin(\phi_c(t)), \quad \omega_c = \frac{d\phi_c(t)}{dt}$
- PM: $u_{PM}(t) = U_c \sin(\phi_{PM}(t)) = U_c \sin(\phi_c(t) + \Delta\phi . f(t))$
  $\phi_{PM}(t) = \phi_c(t) + \Delta\phi . f(t)$
  - where $\Delta\Phi$ is the phase deviation
- FM: $\omega_{FM}(t) = \omega_c + \Delta\omega . f(t) \Rightarrow$

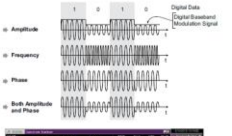
$$\Rightarrow \phi_{FM}(t) = \int_0^t (\omega_c + \Delta\omega . f(x))dx = \omega_c t + \Delta\omega \int_0^t f(x).dx$$

$$u_{FM}(t) = U_c \sin(\phi_{FM}(t)) = U_c \sin\left(\omega_c t + \Delta\omega \int_0^t f(x).dx\right)$$

  - were $\Delta\omega$ is the frequency deviation
  - Index FM: $m = \Delta\omega / \omega_{\mathrm{mod}}$

Simple digital modulations

- Modulating signal is digital data signal (pulse train)
- All basic modulation or their combination can be used
- Basic simple measurements similar to analog modulations.
- Complex measurements require vector signal analyzer



Jitter and phase noise measurement

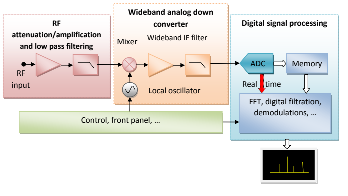
- Oscillators never produce pure sinewave
- Real signal contains amplitude and phase modulation caused by internal instability of oscillator
- Measurement: ratio of a component to carrier (in dBc)

Vector signal analyzer

Only for masters

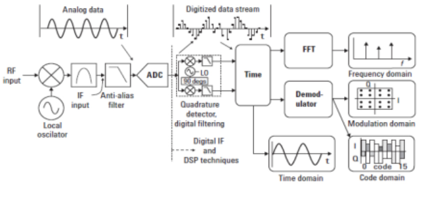
Principle

- Combination of heterodyne filtration with digital signal processing
  - Wideband IF filter (low pass instead of bandpass) - from tens of MHz up to a few hundreds MHz
  - Fast ADC



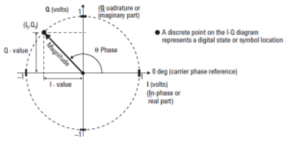
VSA signal processing

- VSA processes signal (sinewave) as vector (real and imaginary components are function of time).
- Results:
  - Magnitude spectrum and measurements as on spectrum analyzer(distortion, bands, components, …)
  - FFT analysis of signal within a frequency band
  - Modulation and code znalysis of digital modulations



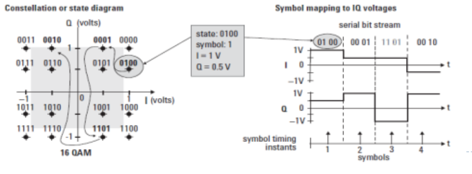
Quadrature amplitude modulation }QAM|

- Instantaneous magnitude and phase can be expressed by a vector in complex plane with:
  - real (In-phase) coordinate and
  - imaginary (Quadrature) coordinate

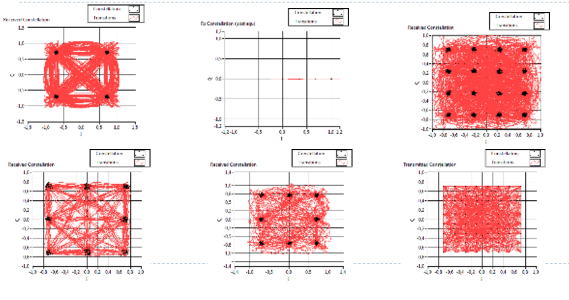


Mapping

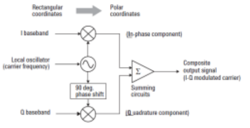
- Binary data value can be combined (1,2,3,4,...bits) and represented by symbols in IQ plane (value - voltage of I and Q for each bit combination).
- Number of needed symbols $2^{\text{number of bits per symbol}}$ = (2, 4, 8, 16, ...)
- Constellation diagram - graphical representation of symbols
- Constellation diagram can be used for comparison of real signal within ideal and for displaying how signals IQ change during data transmission. It can be used for quality assessment of transmitted signal



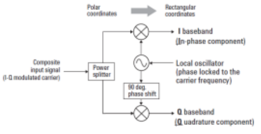
Examples



IQ modulator and demodulator



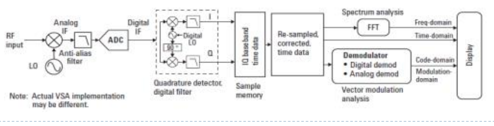
- Modulator controls carrier according to time-variant I and Q signals
- It could be in analog or digital form



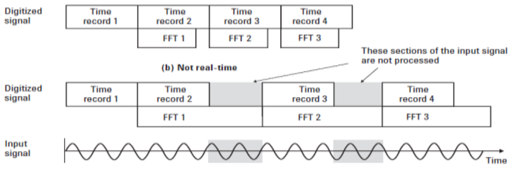
- Demodulator converts the instant position of carrier vector to its coordinates I and Q
- It could be in analog or digital form

Signal processing in VSA

- Correction of errors and imperfections of analog front-end, ADC and quadrature detector in time domain (decimation, resampling, mathematics)
- Frequency domain: windowed FFT
- Code and demodulation domain: demodulation from IQ components, decoding , error analysis, quality of service, ...
- Time analysis of IQ signals (distortion, eye diagram, …)

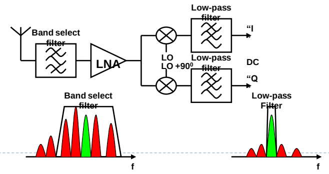


Real time VSA



Homodyne

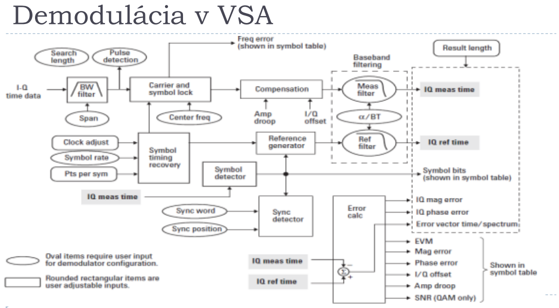
- „homodyne, quadrature downconversion, direct-conversion receiver" processes RF signal without transposing on a IF.
  - Simple and cheap
  - Synchronizing LO with carrier Very difficult.
- Used very rarely



VSA measurements

Only for masters

Demodulácia v VSA



QAM error parameters

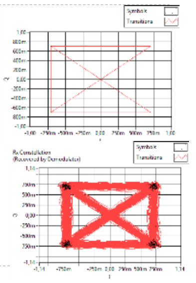
- Error vector magnitude (EVM) – relative magnitude of error vector, which moves real constellation positionvy from the required (ideal) positions to the ideal vector size = relative distance of real position from ideal position to the ideal position

$$EVM_{dB} = 10\log\frac{P_{error}}{P_{reference}}, \quad EVM_{\%} = \sqrt{\frac{P_{error}}{P_{reference}}}100\%$$

$$EVM_{dB} = 20\log\frac{RMS\ (EV\ magnitude)}{RMS\ (the\ most\ distant\ reference\ position)}$$

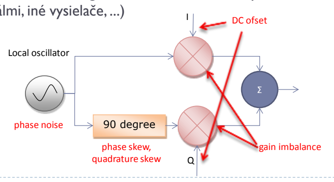
- Modulation error ratio (MER) – similar to EVM except error is related to magnitude of real

$$MER_{dB} = 10\log\frac{P_{error}}{P_{signal}}, \quad MER_{\%} = \sqrt{\frac{P_{error}}{P_{signal}}}100\%$$



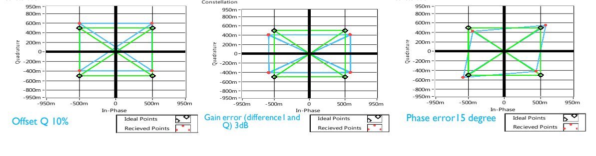
Error sources

- Modulation errors = errors caused by imperfection of IQ modulator (modulation impairments, IQ impairments) and/or generator of I and Q signals (DA convertor)
- Transfer error (additive noise, fading, inter-channel and inter-symbol interference rušenie – medzi kanálmi, iné vysielače, ...)



Modulation error

- Manifestation of modulation errors:



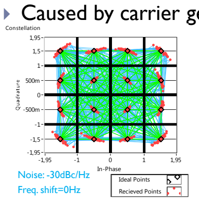
Offset Q 10%

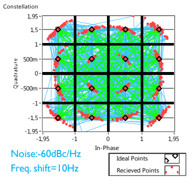
Gain error (difference I and Q) 3dB

Phase error 15 degree

Phase noise and shift

- Caused by carrier generator





ACPR

- ACPR - adjacement channel power ratio is measured for modulated signals:
  - Receiver must be able to reject unwanted signals in the receiving channel (bandwidth)
  - Transmitter must generate and transmit signal in accordance with given standards for generated spectrum to ensure minimal interference with other channels/devices, e.g. limit for ACPR according to FCC for WDMA is -33dBc.
- Different channels spectra covering causes interference, deteriorates received signal quality and increases w kvalitu příjmu a zvyšuje error rate in digital communication
- ACPR is decreased by low pass filtration of IQ signal in transmitters
- ACPR measurement = mask test according to a standard
  - VSA with a wide bandwidth is required, e.g., for channel BW=10MHz, VSA with BW=30MHz is required (minimum)

BER

- Bit Error Rate (BER) dependence on:
  - Receiver quality (SNR)
  - Quality of transmission channel (noise, fading, ...)
  - Interference from sources:
    - in-band
    - Out-of-band

$$BER = \frac{Total\_numb\_of\_error\_bits}{Total\_numb\_of\_transmitted\_bits}$$

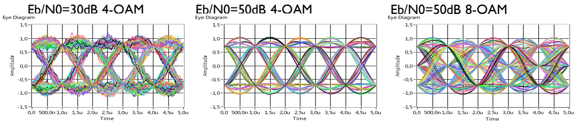
- BER test is performed at defined conditions - interference, number of transmitted bits with stochastic distribution
- Test methods:
  - XOR –delay line for synchronization of transmitted and received bits with following XOR
  - FPGA – delay and comparison in internal structure
  - Digital pin – DSP
  - Digitizer – recording received data stream with off-line processing by software in PC

When BER

- BER is used for other tests:
  - Receiver sensitivity – the limit of minimal input signal given by the receiver input noise
  - Carrier-to-Interference (C/I) – interference of different data transfer iat the same or close frequency
  - In-Band / Out-Band blocking by power harmonic signal without modulation
  - Intermodulation – instead of harmonic signal data transfer producing intermodulation components are used.
  - Maximal sensitivity - ability to receive signal with high power (overloading of receiver input)– opak citlivosti – at given BER.

Eye diagram

- Time waveform of I and Q after demodulation
- Characterization of received signal quality



Some other common parameters and shorts

- AWGN – Additive White Gaussian Noise
- Eb – energy on bit
- N0 – dispersion of AWGN (power)
- Eb/N0 – similar SNR for analog signals but for digital signals. The higher value the better ration signal to noise