SECTION 1

Introduction to the Fourier Integral

1.1 Definition and History

"... a digital signal processor can efficiently compute the Fourier transform and perform specific frequency-domain tasks..."

The scientific and engineering communities have attempted to represent changing signals in two fundamental domains: time and frequency. Temporal changes are easily shown on oscilloscopes, for instance, where change in time is directly proportional to distance across a screen. Representation of signals in terms of frequencies falls under the general category of "spectrum analysis", and has generated a lot of attention recently, due to the increased availability of hardware which makes such representations possible. The first formal approach to spectrum analysis probably dates back to the work of Fourier, who showed how to mathematically represent a general class of time-varying phenomena in terms of sine and cosine functions of particular frequencies. His work is best known as the Fourier Integral (inverse Fourier transform) (see Reference 1):

$$\chi(t) = \int_{+\infty}^{+\infty} X(f)e^{j2\pi ft}dt \qquad \text{Eqn. 1-1}$$

where: $j = \sqrt{-1}$ and $e^{j2\pi ft} = \cos(2\pi ft) + j\sin(2\pi ft)$

When interpreted as an infinite summation, the previous integral is simply a linear combination of a number of sine and cosine functions (expressed by the complex exponential), each one of which is weighted by the complex amplitude X(f). Conversely, the complex frequency function X(f) can be derived from the time-varying signal $\chi(t)$ by the Fourier Transform:

$$X(f) = \int_{+\infty}^{+\infty} \chi(t)e^{-j2\pi ft}dt \qquad \text{Eqn. 1-2}$$

The two expressions shown in Eqn. 1-1 and Eqn. 1-2 define a Fourier transform pair $\chi(t)$ and X(f). The Fourier transform X(f) determines the frequency content of the signal in question, while $\chi(t)$ shows the way the signal varies as a function of time. Note that, in general, $\chi(t)$ can be directly measured (for instance, displayed on an oscilloscope). X(f) remains a mathematical expression which attempts to express our intuitive perception of frequency.

Unfortunately, it is not always true that the concept of frequency, as defined by the Fourier transform in Eqn. 1-2, and the intuitive concept of frequency as we perceive it, are identical. For instance, music consists of tones (frequencies) which vary over time. Although we can clearly perceive time-varying frequencies, Eqn. 1-2 does not allow for Fourier's concept of frequency to have any time-varying character— X(f) is a function of frequency only.

1.2 Use of the Fourier Transform

Because of the basic nature of the frequency concept, practical applications of the Fourier transform are abundant. As more cost-efficient methods become available to compute the Fourier transform, the number of practical solutions to frequency-based problems will grow even larger. In these frequency-based applications, a digital signal processor can efficiently compute the Fourier transform (as defined in ***SECTION 1.1 Definition And History***), and perform specific frequency-domain tasks such as elimination of certain frequency components, etc.

Three general types of Fourier transform applications are:

1. Number-Based — Most spectrum analysis applications require the direct evaluation of the Fourier transform as in Eqn. 1-2. Since the Fourier transform is a mathematical expression, these applications are based on numerical computations, and can be termed number-based. Examples range from spectrum analysis laboratory instrumentation and professional audio equipment to velocity estimation in radar. Note that in number-based applications the accuracy of the computed numbers is of vital importance to the performance of the overall system. For instance, the quality-conscious audio industry requires 16-bit or more precision in order to eliminate audible distortion.

2. Pattern-Based — Many problems involve the recognition and detection of signals with a specific frequency content (a predefined spectral pattern). For instance, speech consists of segments of sound with very specific frequency characteristics. In this type of application, the conversion to the frequency domain is often only a single step in the overall task. It is important that this conversion process be as fast as practical, to allow for sufficient time to perform computationally intensive pattern matching techniques. In addition to providing fast Fourier transform computations, the processor in question needs to be fast at general-purpose DSP tasks so that it can perform a variety of frequency-based calculations for pattern matching.

3. Convolution-Based — The third class of applications of Fourier transforms uses the transform as a simple mathematical tool to perform general filtering in a very efficient manner. This concept is based on the property that the Fourier transform of the convolution of two time-signals:

$$y(t) = \int_{+\infty}^{+\infty} \chi(t-\tau)h(\tau)d\tau \qquad \text{Eqn. 1-3}$$

is equal to the product of the individual transforms:

$$Y(f) = X(f)H(f) \qquad \text{Eqn. 1-4}$$

Eqn. 1-3 (better known as the convolution integral) represents the output of a linear filter with impulse response h(t) and input signal x(t). Clearly, in the frequency domain, the output of a filter can be obtained by a simple multiplication, whereas in the time domain, a more complicated convolution integral needs to be solved. The amount of computation involved in evaluating the integral in Eqn. 1-3 becomes particularly large when the impulse response h(t) has a long time duration which often prevents real-time implementation. Clearly, if the Fourier transform X(f) of the signal can be computed efficiently, the filtering operation itself can be achieved by simple multiplications.

The combined number of computations (for computing the Fourier transform, for filtering in the frequency domain, and for obtaining the inverse Fourier Transform) is often less than the total number of calculations required to compute Eqn. 1-3 directly. This is especially true when the filter in question performs a simple frequency discrimination function (lowpass, bandpass, highpass, bandreject, etc.). In this case, the multiplications in the frequency domain can be replaced by a simple masking operation, which removes the stopbands and leaves the passband(s) unchanged.

Although no direct frequency information is extracted from the signal, the Fourier transform is used as a mathematical tool for fast-filtering applications. Note that again, fast Fourier transform and inverse Fourier transform “engines” are needed in order to provide the real-time filtering operation.

In summary, the basic nature of the frequency concept indicates that the number of possible frequency domain applications is as large as more conventional time domain applications. In the past, frequency domain applications were either difficult to implement or could not be realized in a cost-efficient manner because of the lack of low-cost, high-performance hardware. This application note demonstrates that the DSP56001/2 and the DSP96002 Families of digital signal processors fulfill the demanding requirements imposed by frequency domain problems. In addition to providing a fast implementation of high-precision Fourier transform computations, the general-purpose nature of the instruction set allows for a ***complete, single-chip, low-cost*** integrated solution to a wide variety of frequency domain problems. ■